Syllabus for MAE 700 (Nonlinear and Optimal Control):

Books (recommended): *Geometric Control Theory, (2008, Cambridge Univ. Press),* by V. Jurdjevic; *Nonholonomic Mechanics and Control (2015, Springer),* by A. M. Bloch et al.
Books (optional): *Geometric Control of Mechanical Systems (2004, Springer),* by F. Bullo and A. Lewis; *Control Theory from the Geometric Viewpoint (2004, Springer)*, by A. Agrachev and Y. Sachkov.
Prerequisites: ELE 612 (Modern Control Systems) or equivalent expected; instructor permission
Course content on Blackboard: blackboard.syr.edu

Course Topics

Introduction to geometric control: Lie algebras, distributions, nonlinear controllability and nonlinear observability. Control of simple mechanical systems: geometric mechanics, Lagrangian and Hamiltonian methods. Introduction to optimal control theory: Pontryagin's Maximum Principle applied to systems evolving on manifolds.

Course Contents and Teaching Philosophy

This course is research-oriented, and it will be taught accordingly. The main topics will be covered in the sequence listed above. Introduction to nonlinear (geometric) control will begin with introduction to the concepts of nonlinear controllability and observability. Prior to that, necessary concepts from differential geometry, like coordinate charts, vector fields, distributions and codistributions, will be introduced. Following that, control of mechanical systems will be covered in the framework of geometric mechanics, with an introduction to the configuration manifold, velocity phase space (tangent bundle of configuration manifold) and momentum phase space (cotangent bundle) of mechanical systems. Optimal control on manifolds will be introduced, along with Pontryagin's maximum principle. Reading assignments on topics covered in class will be given. A mid-semester exam will be conducted that will cover basic concepts on nonlinear (geometric) control systems covered in class. A final term paper on analysis of a nonlinear control system of the individual's choice is required for this class. This individual term paper has to be presented in front of the class during the final week of classes for the semester.

Schedule

Week 1: Introduction to differentiable manifolds, coordinate charts, and vector fields (week of Jan 17)
Weeks 2-3: Vector fields, Lie brackets and Lie algebras, introduction to distributions (codistributions) of vector (covector) fields
Week 4: Distributions and codistributions, Frobenius Theorem, introduction to matrix Lie groups

Weeks 5-6: Matrix Lie groups and their Lie algebras, the exponential map, Campbell-Baker –Haussdorff formula, introduction to nonlinear control systems
Week 7: Autonomous control affine systems, systems with drift and drift-free control systems Week 8: Introduction to geometric nonlinear control: controllability and accessibility
Midterm Exam (take-home: given Tuesday, March 7; due Thursday, March 9)
Week 9: Spring break (no classes March 12-19)
Weeks 10-11: Nonlinear observability, observability codistribution; zero dynamics and normal forms; mechanical systems
Weeks 12-13: Controllability concepts for mechanical systems: simple mechanical control systems, configuration manifold and configuration controllability; Hamiltonian and Lagrangian control systems
Week 14: Optimal control problem statement, attainable sets, Hamiltonian vector fields
Week 15: Pontryagin's maximum principle (geometric statement)
Final Project Presentations: during last week of classes (likely May 2).

Additional Reference Material

The material covered in class will not be drawn from any single book; however, material covered in the lectures will be self-contained and adequate to handle the exam, reading assignments and project in the course.
Besides the books listed, any other textbook on nonlinear control systems theory and analysis will also be useful as a reference.